METHODS FOR DETERMINATION OF THE EXCITATION CURRENT IN SYNCHRONOUS GENERATORS

By

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SVK
Methods for determination of the excitation current in synchronous generators

Technical Report

ABSTRACT

Synchronous generators or alternators are synchronous machines used to convert mechanical power to ac electric power.

They are widely used as source of electrical power. Nowadays, they are dominating the power market. Large machines generating electrical power in hydro, nuclear or thermal power stations are the synchronous generators.

Synchronous generators can generate active and reactive power independently and have an important role in voltage control.

The field current in synchronous machine is of great importance. The most realistic value of the excitation current is also very important to the excitation system design. This value could be roughly found by varying the real, reactive power and terminal voltage of the machine. For that purpose a mathematical relation between the field current of the synchronous generator, active and reactive power, terminal voltage, and the machine parameters (Xd, Xq, Xp) is found.

Theoretical calculations using non-saturated and saturated reactances are performed showing a considerable difference in the excitation current values. Comparison with experimental data was done.
ACKNOWLEDGMENTS

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PREFACE

The aim of this master thesis work has been to study the field current in a synchronous generator in both unsaturated and saturated case. A short description of magnetic material is found in the first chapter since synchronous generators deal with such materials, which are used to shape and direct the magnetic fields that act as a medium in the energy conversion process.

In the second chapter an introduction about synchronous generators types and equivalent circuits is included. The third chapter discuss the operation of the synchronous generators under unsaturated and saturated conditions.

The reader who wants to go straight into the heart of the thesis work is advised to skip the first two chapters.
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I-INTRODUCTION

Synchronous generator is the absolutely dominating generator type in power systems. It can generate active and reactive power independently and has an important role in voltage control. The synchronizing torques between generators act to keep large power systems together and make all generator rotors rotate synchronously. This rotational speed is what determines the main frequency, which is kept very close to the nominal values 50 or 60 Hz.

Synchronous generators come with round (cylindrical) rotor or with salient pole rotor. The number of poles (magnetic N and S poles in the rotor) ranges from two to almost one hundred.

Since magnetic materials form a major part in electrical machines, saturation effects shall be considered. Current work in the field of electrical machines is concentrated on how saturation in the magnetic materials is affecting the synchronous reactance of the generator and hence the field current. For that purpose, both saturated and unsaturated case were studied, each one apart. Variations in the field current could be noticed as well.

In this thesis a 55MVA, 16.5KV salient pole synchronous generator was studied. This generator is located in Älvekarleby a hydro power station. The tests performed have proved very good compliance between calculated and measured parameters.
II-MAGNETIC CIRCUITS

This project is concerned with the study of synchronous generators, a device that convert mechanical energy into electrical energy. In such device magnetic materials are used to shape and direct the magnetic fields that act as a medium in the energy conversion process. A major advantage of using magnetic material in electrical machines is the fact that high flux density can be obtained in the machine, which results in large torque or large machine output per unit machine volume. Hence the size of the machine is reduced by the use of magnetic materials. In this chapter we will point on such materials. Since magnetic materials form a major part in electrical machines, we will point on such materials in this chapter.

II.1 Magnetic circuit

In a rotating machine, i.e. synchronous machines, ferromagnetic materials in conjunction with an air gap form the magnetic circuit.

II.1.1 Ampere’s circuit law

When a conductor carries current a magnetic field (or flux) is generated around it (figure II.1). The relationship between current and field intensity can be obtained by using Ampere’s law, which states that the line integral of the magnetic field intensity $H$ around a closed path is equal to the total current.

\[ \oint H \cdot dl = \sum i \]

Where $H$ (A/m): magnetic field intensity.

\[ i \] (A): current

Figure II.1: Magnetic field around a current-carrying conductor
II.1.2 B-H relation

The magnetic field intensity $H$ is linked to the magnetic flux density $B$.

For a certain value of the permeability $\mu$, $H$ is related to $B$ by:

$$B = \mu H$$

where $B$ (Weber/m$^2$, or Tesla): Magnetic flux density

$H$ (A/m): Magnetic field intensity.

$\mu$: Permeability of the medium

The permeability $\mu$ consists of two components $\mu_0$ and $\mu_r$. $\mu_0$ is the permeability of free space and is equal to $4\pi 10^{-7}$, and $\mu_r$ is the relative permeability of the medium and varies in the range of 2000 to 6000 for materials used in electrical machines (synchronous generators). Note that $\mu_r = \mu_r(B)$ for these materials, i.e. not a constant.

II.1.3 Magnetic equivalent circuit

In figure II.2 below a simple magnetic circuit, consisting of a toroid. When a current $i$ flows through the coil of $N$ turns, magnetic flux will be created in the core material. The flux outside the toroid is called leakage flux. If the coil is tightly wound the leakage flux is small, and could be neglected.

![Toroid magnetic circuit](image)

Assuming that all fluxes are confined in the toroid, i.e. the leakage flux is zero, then the flux crossing the cross section ($A$) of the toroid is:

$$\phi = \int \vec{B}.dA = \{ B \text{ is constant over the cross section} \} = BA \text{ (wb)}$$

The inductance $L$ and the reluctance $\mathcal{R}$:

$$\phi = \frac{\mu Ni}{l} A = \frac{Ni}{l/\mu A} = \frac{Ni}{\mathcal{R}} = \frac{F}{\mathcal{R}}$$
\[ L = \frac{1}{\Re} \] and the reactance \( X = wL \)

Where \( F \) is the magnetomotive force (Mmf).

So as the reluctance \( \Re \) increases, the flux \( \phi \) will decrease and the reactance \( X \) will decrease as well.

Figure II.3 shows a magnetic equivalent circuit that represent the magnetic circuit of the toroid.

![Figure II.3: Equivalent magnetic circuit](image)

**II.1.4 Magnetization curve**

If the magnetic intensity \( (H) \) in the core is increased, the flux density \( (B) \) will increase as shown in figure II.4. \( B \) increases almost linearly for low values of \( H \). However for higher values of \( H \), the relation between \( B \) and \( H \) becomes nonlinear. The reluctance \( \Re \) is dependant on \( B \) as \( \mu_r \) depends on \( B \). It is low when \( B \) is low and high when \( B \) is high. The plot of \( B \) versus \( H \) is called the magnetization curve and is illustrated in figure II.4.

![Figure II.4: Saturation curve](image)


Synchronous generators are by definition synchronous, meaning that the electrical frequency produced is locked in or synchronized with the mechanical rate of rotation of the generator. Synchronous generators or alternators are synchronous machines used to convert mechanical power to ac electric power. Synchronous machines are used widely as generators of electrical power. Nowadays, they are dominating the power market. Large machines generating electrical power in hydro, nuclear or thermal power stations are the synchronous generators. They can generate active and reactive power and has an important role in voltage control. A Synchronous machine is a doubly excited machine. Its rotor poles are excited by a dc current and its stator windings are connected to the ac-supply. In a synchronous generator, a dc current is applied to the rotor winding, which produces a rotor magnetic field. The rotor of the generator is then turned by a prime mover, producing a rotating magnetic field within the machine. This rotating magnetic field induces a three-phase set of voltages within the stator winding of the generator. The dc current applied to the rotor is achieved by two common approaches:

1- Supply the dc power from an external dc source to the rotor by means of slip rings and brushes.
2- Supply the dc power from a special dc power source mounted directly on the shaft of the synchronous generator.

Synchronous machines can be divided into two groups:
1- High speed machines with cylindrical (or non salient pole) rotors.
2- Low speed machines with salient poles rotors.

The cylindrical or non-salient pole rotor has uniform air gap. The rotor is long and has a small diameter. The number of pole is two or sometimes four. Salient pole generator has a larger number of poles than cylindrical rotor, a nonuniform air gap and operates at lower speeds. They are normally used in hydroelectric power plants and water turbines drive them.

Figure III.1 shows the basic structure of a three-phase salient and non-salient pole synchronous machine.

![Figure III.1: Basic structure of a three phase synchronous machine](image-url)
III.1 Equivalent circuit model

Developing an equivalent circuit for a machine is an important matter. Since such circuit could be used to study the performance of the machine with accuracy. Assuming steady state conditions then the circuit time constants of the field and damper windings need not to be considered. The equivalent circuit will be derived for positive sequence and is shown in figure III.2.

The current $I F$ in the field winding produces a flux $\phi_f$ in the air gap. The flux $\phi_a$ in the stator winding produces the current $I a$. Part of it is $\phi al$ the leakage flux, which links with the stator winding and a major part is $\phi ar$ the armature reaction flux which links with the field winding.

\[ X_{ar} \] is the armature reactance, which accounts for $\phi ar$ and $Xal$ is leakage reactance for $\phi al$. The stator resistance $Ra$ is neglected. If those two reactances are combined into one reactance then we get what we call the synchronous reactance to be: $Xs= Xar+Xal$. This synchronous reactance $Xs$ takes into account all the flux (magnetizing and leakage). $Xar$, $Xal$ and $Xs$ are called machine variables and depend on the size of the machine.

III.2 Salient pole synchronous generator- unsaturated case

III.2.1 Introduction

Low speed synchronous generators have salient poles and nonuniform air gaps. The magnetic reluctance is low along the poles and high between the poles. So, an mmf will produce more flux if it is acting along the pole axis, called the $d$-axis, and less flux if it is acting along the quadrature axis, called the $q$-axis.

Because of that, the magnetizing reactance $Xar$ is not unique, but depends on the load angle ($\delta$). Detailed explanation and figures are shown underneath.

In figure III.3 (a), the stator current $Ia$ is in phase with the excitation voltage $Ef$. The field mmf $Ff$ and flux $\phi f$ are along the $d$-axis and the armature mmf $Fa$ and flux $\phi ar$ are along the $q$-axis. In figure III.3 (b), the stator current ($Ia$) is considered to lag $Ef$ by $90^\circ$.

The armature mmf $Fa$ and flux $\phi ar$ are along the $d$-axis opposing the field mmf $Ff$ and flux $\phi f$. One can notice that the magnetization reactance is more if $Ia$ lags $Ef$ than if $Ia$ is in phase with $Ef$. Therefore, $Xar$ depends on the power factor of the stator current ($Ia$).
The armature mmf $F_a$ and the stator current $I_a$ are made of two components respectively $(F_d, F_q)$ and $(I_d, I_q)$. They produce flux $(\psi_{ad}, \psi_{aq})$ and those fluxes could be represented by the following armature reactances (one along the $d$ axis and one along the $q$ axis), $X_{ad}$ and $X_{aq}$.

As a result the synchronous reactance $X_s$ has two components: $X_d$ and $X_q$. Introducing the leakage reactance $(X_{al})$ and assuming that $X_{al}$ is the same in $d$ and $q$ direction then:

$$X_d = X_{ad} + X_{al} \quad \rightarrow d\text{-axis synchronous reactance}$$
$$X_q = X_{aq} + X_{al} \quad \rightarrow q\text{-axis synchronous reactance}$$

It is obvious that $X_d > X_q$ because the reluctance in the $q$ axis is higher than the $d$ axis.
Typically, $X_q \approx 0.65 X_d$ in hydro generators.

Figure III.4 shows a graphical representation for different components ($F_a$, $I_a$, $\phi a r$).

III.2.2 Equivalent circuit and phasor diagram

Examining the equivalent circuit and the phasor diagram in figures III.5 for a salient pole synchronous generator, one can derive the following mathematical relations:

\[
\begin{align*}
E_f &= \bar{V}_t + I_d X_d + I_q j X_q \\
\bar{V}_t &= \text{Re}(V_t) + j \text{Im}(V_t) \\
I_a &= I_d + I_q \\
I_d &= |I_a| \sin(\phi + \delta) \\
I_q &= |I_a| \cos(\phi + \delta) \\
tan \delta &= \frac{|I_a| X_q \cos \phi}{\bar{V}_t + |I_a| X_q \sin \phi}
\end{align*}
\]  

Where $\phi$ and $\delta$ are the power factor and power angle respectively.
### III.2.3 Power transfer

A synchronous generator can generate active and reactive power independently.

The complex power per phase is:

\[ S = \frac{V_I I_a}{X_d} = \frac{V_I \angle - \delta (I_q - j I_d)}{X_d} \]  

\[ S = |S| e^{-j \delta} \]  

From the above phasor diagram:

\[ I_d = \frac{|E_f| |V_I| \cos \delta}{X_d} \]  

\[ I_q = \frac{|V_I| |E_f| \sin \delta}{X_q} \]  

Substituting (1) and (2) into (3) \( S \), then:

\[ S = \frac{|V_I|^2 \sin \delta}{X_q} - \delta - \frac{|V_I|^2 |E_f|^2 \cos \delta}{X_d} - 90^\circ - \delta = P + jQ \]  

Where \( P \) (MW) and \( Q \) (MVAR) are the real and reactive power generated respectively. The real power \( P \) is made of two terms; The first term \( P_f \) (same as in cylindrical rotor generator) due to the field excitation and \( P_r \) power due to saliency of the machine.

\[ P = |V_I| |E_f| \sin \delta + \frac{|V_I|^2 (X_d - X_q)}{2 X_d X_q} \sin 2 \delta = P_f + P_r \]  

\[ Q = \frac{|V_I|^2 |E_f|^2 \cos \delta}{X_d} - \frac{|V_I|^2 \sin^2 \delta + \cos^2 \delta}{X_q} \]  

\[ |S| = |P + jQ| \]  

From III.11, III.12 and III.13 \( E_f \) could be found since it is the only unknown variables. Knowing \( E_f \), then \( I F = E_f \) since it is in pu.

### III.2.4 Illustrative Example

For the following base values:

- \( U_n = 9.8 \) (Nominal voltage in kV)
- \( S_n = 60 \) (Nominal complex power in MVA)
- \( I_n = S_n / U_n / \sqrt{3} \) (Nominal current in kA)
- \( p_f = 0.85 \) (Power factor)
- \( Z_b = U_n^2 / S_n \) (Base Impedance)
And at an operating point:

\[
\begin{align*}
X_d &= 0.89 \quad \text{(d axis synchronous reactance)} \\
X_q &= 0.65 \times 0.89 \quad \text{(q axis synchronous reactance)} \\
p_f &= 0.819 \quad \text{(Power factor at an operating point)} \\
p_h &= \arccos(p_f) \quad \text{(Power factor angle)} \\
V_t &= 10.586/\text{Un} \quad \text{(Terminal voltage at operating point)} \\
l_a &= (3.295/\text{In}) - p_h \quad \text{(Stator current at operating point)} \\
P &= 0.8247 \quad \text{(Real Power pu)} \\
Q &= 0.5778 \quad \text{(Reactive Power pu)}
\end{align*}
\]

The phasor diagram for a salient pole synchronous generator is shown in figure III.6, where \( E_f = I_f = 1.6890 \text{pu} \)

![Figure III.6: Phasor diagram for the unsaturated case](image)

The power angle \( \delta \) is an important parameter in a synchronous generator. Its characteristics are shown in figure III.7. Where \( P, P_f \) and \( P_r \) versus \( \delta \) are plotted. We can notice that the resultant power \( P \) is higher than that of a cylindrical rotor generator and it occurs at \( \delta \) less than 90°. This makes the generator respond quickly to changes in shaft torque.
A family of power angle characteristics at different values of excitation and constant terminal voltage is shown in figure III.8. If the field excitation (\(E_f\)) is reduced to zero the salient pole generator still develop power (torque) because of the saliency of the rotor structure which is not the case in a cylindrical rotor synchronous generator where \(P=0\) for \(E_f=0\). Moreover, when \(E_f=0\) and for \(\delta=45^\circ\), \(P\) reaches its maximum. We can also notice that as the real power (\(P\)) increases the excitation voltage (\(E_f\)) increases.

Figure III.8: Real power versus \(\delta\) for \(E_f=0, 0.5, 1, 1.5\).
It was from our interest to find a relation between the excitation voltage or current and the voltage, power angle, real and reactive power of a salient pole synchronous generator (i.e. hydro generator). To solve this problem more machines parameters ($X_d$, $X_q$) shall be introduced. For that purpose MATLAB software was used. The above plots were generated by MATLAB as well.

### III.3 Salient pole synchronous generator - Saturated case

#### III.3.1 Introduction

When the field current $IF$ flows through the rotor field winding, it creates a flux in the air gap. If the rotor is now rotated by the prime mover (turbine...) a rotating field is produced in the air gap. This field is called the excitation field because it is produced by the field current ($IF$). The alternating flux will induce voltages in the stator windings.

The rotor speed and frequency of the induced voltage are related by:

\[
n = \frac{120f}{p}
\]

Where $n$ is the rotor speed in rpm

$p$ is the number of poles

The field voltage is related to the flux by:

\[
E_f = 4.44f \phi_i NK
\]

where $\phi_i$ is the flux per pole due to the excitation current $IF$

$N$ is the number of turns in each phase

$K$ is the winding factor

The excitation or field voltage is proportional to the speed of the machine and to the excitation flux as well.

\[
E_f \propto n \phi_i
\]

The variation of the excitation voltage with the field current is shown in figure III.9. At $IF=0$ there is a small induced voltage due to the residual magnetism. Initially the voltage rises linearly with the field current, but as the field current is further increased the flux $\phi_i$ does not increase proportionally with $IF$ because of saturation of the magnetic circuit, and therefore $E_f$ does not increase linearly with $IF$ and it levels off.
The field current or excitation required to operate a synchronous machine may be obtained by the method described below. To make these calculations, machine information's are required: Open circuit, and short circuit characteristics curve, armature resistance, unsaturated quadrature axis reactance, and the Potier or leakage reactance. Methods for determining the Potier or leakage reactance [IEEE Std 115-1995(R2002)] are described in the following clauses.

### III.3.2 Determination of the synchronous reactance

The synchronous reactance is an important parameter in the equivalent circuit of a synchronous generator. It can be determined by performing two tests, an open circuit test and a short circuit test.

**Open circuit test:**

The synchronous generator is driven at the synchronous speed, and the open circuit terminal voltage is measured as the field current $IF$ is varied. The curve showing the variation of $Ef$ with $IF$ is known as the open circuit characteristic (OCC). As the field current is increased, the magnetic circuit shows saturation effects. The line passing through the linear part of the OCC is the air gap line.
Methods for determination of the excitation current in synchronous generators

Figure III.10: Testing Circuit for the open circuit test

Short circuit test:

The synchronous generator is driven at the synchronous speed. The field current $IF$ is measured with an ammeter as well the armature currents. The variation of the armature current with the field current is known as the short circuit characteristic (SCC). SCC is a straight line this is because when the terminals of the machine are short-circuited the magnetic circuit doesn’t saturate because the air gap flux remains at low level.

Figure III.11: Testing Circuit for the short circuit test

Figure III.9 shows the open circuit, air gap line and short circuit curves. From the air gap line voltage and the short circuit line current of the generator we can obtain the d-axis synchronous reactance.

From figure III.9 and for $IF=1$pu $Isc=1.13$ pu and $Ef=1$ so the d-axis synchronous reactance would be $=Ef/Isc=1/1.13=0.88$ pu.

So $Xd=0.88$ pu, hence $Xq\approx 0.65Xd\approx 0.572$

III.3.3 Potier reactance determination under normal machine operation

This method is applicable when the machine is operating near full load and with terminal conditions at unity power factor.

Readings are taken of armature voltage, real power, reactive power and field current. The direct-axis synchronous impedance ($Xd$) must be known, as well as the open circuit saturation curve. For salient pole machines, the quadrature axis synchronous reactance ($Xq$) must also be known.

The following steps outline the procedure for determining a per unit value of $Xp$.

a) Calculate a pu value of field current ($IFU$) at an operating point. $IFU$ is determined by locating $EfU$ on the air gap line and neglecting the stator resistance ($Ra$).
The following data are given for a 3 φ, 55 MVA, 16.5KV synchronous generator in Älvkarleby hydro power station.

\[ Sn: 55 \text{ MVA} \]
\[ Un: 16.5 \text{ Kv} \]
\[ In: 1.925 \text{ KA} \]
\[ Cosfin: 0.9 \]

At Operating Point:
\[ Ubp: 17.148 \text{ Kv} \]
\[ Ibp: 1.8110 \text{ KA} \]
\[ Cosfibp: 0.8979 \]
\[ IFbp: 0.9925 \text{ A} \]

\[
\tan\delta = \frac{Ibp \cdot Xq \cdot \cos\phi}{Ubp + Ibp \cdot Xq \cdot \sin\phi}
\]

\[
Id = Ibp \cdot \sin(\phi + \delta)
\]
\[
Iq = Ibp \cdot \cos(\phi + \delta)
\]
\[
Efu = Ubp + jIqXq + IdXd
\]

The phasor diagram for unsaturated generated voltage \( Efu \) of a salient pole synchronous generator is shown in figure III.12:

The phasor diagram for unsaturated generated voltage \( Efu \) of a salient pole synchronous generator is shown in figure III.12:

\[
Efu = IFU = 1.5894 \text{ pu}
\]

b) Determine the p.u value of the measured field current \( IF \) by dividing that current by the base value of field current corresponding to 1.0 per unit terminal voltage on the air-gap line of the given open circuit saturation curve. And \( IF \) would be equal to 1.8273 pu

c) Determine \( IFS = IF - IFU = 0.2379 \text{ pu} \). Refer to figure III.9 for more details.
d) Using any desired fitting process; determine the p.u value of $U_p$ (the voltage behind Potier reactance) on the ordinate. By using the difference ($IFS$) between a voltage value on the open circuit saturation curve and the same voltage value on the air gap line. $U_p$ is represented by a line parallel to the x-axis and is calculated to be 1.1621 pu. Refer to figure III.9 for more details.

e) The phasor position of $U_p$ relative to $U_b$ is not known; however, figure III.13 indicates the actual phase relationship between $U_b$ in per unit and $I_a$ in per unit. The power factor angle $\phi$, is also shown.

![Figure III.13: Calculation of magnitude $U_p-U_b$](image)

f) The per unit magnitude of the phasor $E_p-E_a$ can now be determined by the following equation:

$$|U_p - U_{bp}| = \sqrt{U_p^2 - (U_{bp} \cos \phi)^2} - U_{bp} \sin \phi = 0.2352 \text{ pu}$$

Then $X_p = \frac{|U_p - U_{bp}|}{|I_q|} = 0.25 \text{ pu}$

### III.3.4 Field current determination

Once $X_p$ is found, then the field current could be calculated as follows:

- First the real and reactive power as well the stator voltage are considered to be input data:
  
  \[ P_n = 0.013 \text{ pu} \]
  \[ Q_n = 0.096 \text{ pu} \]
  \[ U_b = 0.997 \text{ pu} \]

- Second the power factor $\cos \phi = \frac{P_n}{\sqrt{P_n^2 + Q_n^2}} = 0.1299$ is calculated as well the power factor angle $\phi = \cos^{-1}(\phi) = 1.4406$. 

-Third and from $P_n$, $Q_n$ and $U_b$ one can calculate $I_b$

$$I_b = \frac{\sqrt{P_n^2 + Q_n^2}}{U_b} = 0.0973$$

-Fourth, the power angle $\delta = \tan^{-1}\left(\frac{I_{bp}.X_q \cos \phi}{U_{bp} + I_{bp}.X_q \sin \phi}\right) = 0.0070$ is found.

-Fifth, the currents in all $(d, q)$ directions are calculated:

$$I_a = I_b(\cos \phi - j \sin \phi) = 0.0126 - 0.0965j \text{ pu}$$

$$I_d = I_b \sin(\phi + \delta) = 0.0966 \text{ pu}$$

$$I_q = I_b \cos(\phi + \delta) = 0.0120 \text{ pu}$$

-Sixth, the voltage behind the Potier reactance is found: $|U_p| = |U_b + jI_aX_p| = 1.0213 \text{ pu}$. Note that the Potier reactance $X_p$ is found in III.3.3.

-Seventh, the field voltage under no saturation ($E_{fn} = 1.0839 \text{ pu}$) is calculated as before.

-Eight, the saturation increment current ($IFS = 0.0888$) is found from the both OCC and air gap line curves and the field current is found to be equal to $E_{fn} +$ saturation increment current $= 1.0839 + 0.0888 = 1.1727 \text{ pu}$.

To simplify the field current calculations, a matlab code was written. In this code the function $IFcall$ calculate the field current and draw the phasor diagram of a salient pole synchronous machine (Saturated case)

$IFcall(X_d, X_q, X_p, x, P_n, Q_n, U_b)$

The first three input arguments are respectively, the $d$, $q$ axis synchronous reactances and the Potier reactance.  
The fourth input argument correspond to $x(3)$=exponent $x(4)$=factor 
The fifth and the sixth input arguments correspond to the real and reactive power (MW, Mvar) respectively 
The seventh input argument is the terminal voltage (KV)

A menu will be displayed where you have to choose: Value for $IF$ or Phasor Diagram.

For different values for $P$, $Q$ and $U_b$, $IF$ as a function of $X_p$ is calculated and the final results are illustrated in tableIII.1 and tableIII.2. Those two tables could be read as follow: For $P=0.013$, $Q=0.096$ and $U_b=0.997$, $X_p$ was found to be equal to 0.207. Now, fixing $X_p=0.207$ and varying $P$, $Q$ and $U_b$ as it is shown in tableIII.1, then different values for $IFFinal$ could be recorded: $IFFinal=[1.170, 1.777, 1.192, 1.680, 1.288, 1.705, 1.437, 1.883]$
Table III.1: \( P, Q \) and \( Ub \)

<table>
<thead>
<tr>
<th>Xd</th>
<th>Xq</th>
<th>Xd</th>
<th>P</th>
<th>Q</th>
<th>Ub</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.898</td>
<td>0.584</td>
<td>0.207</td>
<td>0.013</td>
<td>0.096</td>
<td>0.997</td>
</tr>
<tr>
<td>0.898</td>
<td>0.584</td>
<td>0.186</td>
<td>0.011</td>
<td>0.564</td>
<td>1.054</td>
</tr>
<tr>
<td>0.898</td>
<td>0.584</td>
<td>0.047</td>
<td>0.228</td>
<td>0.083</td>
<td>1.009</td>
</tr>
<tr>
<td>0.898</td>
<td>0.584</td>
<td>0.187</td>
<td>0.228</td>
<td>0.466</td>
<td>1.055</td>
</tr>
<tr>
<td>0.898</td>
<td>0.584</td>
<td>0.199</td>
<td>0.444</td>
<td>0.130</td>
<td>1.009</td>
</tr>
<tr>
<td>0.898</td>
<td>0.584</td>
<td>0.200</td>
<td>0.440</td>
<td>0.461</td>
<td>1.052</td>
</tr>
<tr>
<td>0.898</td>
<td>0.584</td>
<td>0.252</td>
<td>0.846</td>
<td>0.121</td>
<td>1.008</td>
</tr>
<tr>
<td>0.898</td>
<td>0.584</td>
<td>0.223</td>
<td>0.845</td>
<td>0.501</td>
<td>1.058</td>
</tr>
<tr>
<td>0.898</td>
<td>0.584</td>
<td>0.250</td>
<td>0.878</td>
<td>0.430</td>
<td>1.039</td>
</tr>
</tbody>
</table>

Table III.2: Change of \( IF \) as a function \( Xp \).

<table>
<thead>
<tr>
<th>Xp</th>
<th>( 1.170 )</th>
<th>( 1.777 )</th>
<th>( 1.192 )</th>
<th>( 1.680 )</th>
<th>( 1.288 )</th>
<th>( 1.705 )</th>
<th>( 1.437 )</th>
<th>( 1.883 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.207</td>
<td>1.168</td>
<td>1.761</td>
<td>1.191</td>
<td>1.667</td>
<td>1.285</td>
<td>1.692</td>
<td>1.432</td>
<td>1.865</td>
</tr>
<tr>
<td>0.047</td>
<td>1.159</td>
<td>1.672</td>
<td>1.182</td>
<td>1.597</td>
<td>1.270</td>
<td>1.621</td>
<td>1.412</td>
<td>1.776</td>
</tr>
<tr>
<td>0.187</td>
<td>1.168</td>
<td>1.762</td>
<td>1.191</td>
<td>1.668</td>
<td>1.285</td>
<td>1.692</td>
<td>1.432</td>
<td>1.866</td>
</tr>
<tr>
<td>0.198</td>
<td>1.169</td>
<td>1.771</td>
<td>1.192</td>
<td>1.675</td>
<td>1.287</td>
<td>1.700</td>
<td>1.435</td>
<td>1.876</td>
</tr>
<tr>
<td>0.200</td>
<td>1.169</td>
<td>1.771</td>
<td>1.192</td>
<td>1.675</td>
<td>1.287</td>
<td>1.700</td>
<td>1.435</td>
<td>1.877</td>
</tr>
<tr>
<td>0.252</td>
<td>1.173</td>
<td>1.814</td>
<td>1.196</td>
<td>1.709</td>
<td>1.294</td>
<td>1.735</td>
<td>( 1.447 )</td>
<td>1.925</td>
</tr>
<tr>
<td>0.223</td>
<td>1.171</td>
<td>1.790</td>
<td>1.193</td>
<td>1.690</td>
<td>1.290</td>
<td>1.715</td>
<td>1.440</td>
<td>1.897</td>
</tr>
</tbody>
</table>
IV-Measurements and Results

In order to verify the theoretical results a field measurement was performed. For that purpose and with the aid of Mr. Karl-Olof Jonsson, Dr. Niclas Schönborg (from Svenska Kraftnät) and people from Vattenfall, we went to the hydro power station in Älvkarleby for an experimental test.

From the control room in Älvkarleby hydro power station, $P$, $Q$ and $U_b$ was recorded and field current readings from a multimeter connected to the field winding of the machine are taken as well.

For different active, reactive power and different terminal voltage (stator voltage) the field current was recorded and the results are illustrated in table IV.1. Note that all results are in pu system.

Table IV.1: Measurements done in Älvkarleby hydro station

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$U_b$</th>
<th>$I_{f,meas}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.013</td>
<td>0.096</td>
<td>0.997</td>
<td>1.170</td>
</tr>
<tr>
<td>0.011</td>
<td>0.564</td>
<td>1.054</td>
<td>1.761</td>
</tr>
<tr>
<td>0.228</td>
<td>0.083</td>
<td>1.099</td>
<td>1.182</td>
</tr>
<tr>
<td>0.228</td>
<td>0.466</td>
<td>1.055</td>
<td>1.668</td>
</tr>
<tr>
<td>0.444</td>
<td>0.130</td>
<td>1.099</td>
<td>1.287</td>
</tr>
<tr>
<td>0.440</td>
<td>0.461</td>
<td>1.052</td>
<td>1.700</td>
</tr>
<tr>
<td>0.846</td>
<td>0.121</td>
<td>1.008</td>
<td>1.447</td>
</tr>
<tr>
<td>0.845</td>
<td>0.501</td>
<td>1.058</td>
<td>1.897</td>
</tr>
</tbody>
</table>

For different values for $P$, $Q$ and $U_b$, $I_f$ as a function of $X_p$ is calculated theoretically (using the matlab program) and the final results are illustrated in table IV.2.

Table IV.2: $I_f$ as function of $P$, $Q$, $U_b$ and $X_p$

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$U_b$</th>
<th>$X_p$, $I_f$, Final</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.013</td>
<td>0.096</td>
<td>0.997</td>
<td>0.207, 1.17, 1.777, 1.192, 1.68, 1.288, 1.705, 1.437, 1.883</td>
<td>-0.73 -0.76</td>
</tr>
<tr>
<td>0.011</td>
<td>0.564</td>
<td>1.054</td>
<td>0.186, 1.168, 1.761, 1.191, 1.667, 1.285, 1.692, 1.432, 1.865</td>
<td>-0.12 0.74 -0.04 -0.13 -0.49 -1.03 -1.68</td>
</tr>
<tr>
<td>0.228</td>
<td>0.083</td>
<td>1.099</td>
<td>0.047, 1.159, 1.673, 1.182, 1.597, 1.27, 1.621, 1.412, 1.776</td>
<td>-0.89 -4.99 0 -4.23 -1.33 -4.64 -2.46 -6.37</td>
</tr>
<tr>
<td>0.228</td>
<td>0.466</td>
<td>1.055</td>
<td>0.187, 1.168, 1.762, 1.191, 1.668, 1.285, 1.692, 1.432, 1.866</td>
<td>-0.11 0.05 0.74 0 -0.12 -0.45 -1.02 -1.63</td>
</tr>
<tr>
<td>0.444</td>
<td>0.130</td>
<td>1.099</td>
<td>0.199, 1.169, 1.771, 1.192, 1.675, 1.287, 1.7, 1.435, 1.876</td>
<td>-0.04 0.56 0.81 0.42 0 -0.02 -0.85 -1.11</td>
</tr>
<tr>
<td>0.444</td>
<td>0.461</td>
<td>1.052</td>
<td>0.2, 1.169, 1.771, 1.192, 1.675, 1.287, 1.7, 1.435, 1.877</td>
<td>-0.04 0.59 0.82 0.45 0.01 0 -0.84 -1.08</td>
</tr>
<tr>
<td>0.846</td>
<td>0.121</td>
<td>1.008</td>
<td>0.252, 1.173, 1.814, 1.196, 1.709, 1.294, 1.735, 1.447, 1.925</td>
<td>0.27 3.03 1.13 2.44 0.57 2.05 0 1.45</td>
</tr>
<tr>
<td>0.845</td>
<td>0.501</td>
<td>1.058</td>
<td>0.223, 1.171, 1.79, 1.193, 1.69, 1.29, 1.715, 1.44, 1.897</td>
<td>0.09 1.65 0.96 1.31 0.25 0.89 -0.48 0</td>
</tr>
</tbody>
</table>

Comparing the two tables we can notice that in table IV.2 the diagonal elements are the same as the values of $I_f$ in table IV.1. Which means that the measured values for $I_f$ are accurate.
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Methods for determination of the excitation current in synchronous generators

and they are the same as the theoretical values. The relative error (%) for the field current is calculated as follow: 
\[
\%\text{error} = \frac{\text{IFFinal} - \text{IFmeas}}{\text{IFmeas}}
\]
and the results are shown in table IV.2.

For small value of \(X_p=0.047\), the error is quite large since \(X_p\) is proportional to \(U_p\) that could not be accurate in the low saturated interval.

As it was previously mentioned, in hydro power stations \(X_q=0.65Xd\). In order to see how a small change in the reactances (\(Xd\) or \(X_q\)) affect \(IF\), \(X_q\) was varied. It was noticed that for different \(X_q\)'s (\(X_q=0.5Xd\), \(0.6Xd\), \(0.7Xd\)) the error compared to the case \(X_q=0.65Xd\) is increased. So, \(X_q=0.65Xd\) is the best approximation. Tables IV.3, IV.4 and IV.5 show the results.

- For \(X_q=0.5 Xd\)

Table IV.3: \(IF\) as function of \(P\), \(Q\), \(Ub\) and \(X_p\)

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>Ub</th>
<th>(X_p)</th>
<th>(IF_{final})</th>
<th>(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.013</td>
<td>0.096</td>
<td>0.997</td>
<td>0.207</td>
<td>1.17</td>
<td>1.777</td>
</tr>
<tr>
<td>0.011</td>
<td>0.564</td>
<td>1.054</td>
<td>0.186</td>
<td>1.17</td>
<td>1.761</td>
</tr>
<tr>
<td>0.228</td>
<td>0.083</td>
<td>1.009</td>
<td>0.086</td>
<td>1.16</td>
<td>1.694</td>
</tr>
<tr>
<td>0.228</td>
<td>0.466</td>
<td>1.055</td>
<td>0.189</td>
<td>1.17</td>
<td>1.763</td>
</tr>
<tr>
<td>0.444</td>
<td>0.130</td>
<td>1.009</td>
<td>0.254</td>
<td>1.17</td>
<td>1.816</td>
</tr>
<tr>
<td>0.440</td>
<td>0.461</td>
<td>1.052</td>
<td>0.207</td>
<td>1.17</td>
<td>1.777</td>
</tr>
<tr>
<td>0.846</td>
<td>0.121</td>
<td>1.008</td>
<td>0.334</td>
<td>1.18</td>
<td>1.892</td>
</tr>
<tr>
<td>0.845</td>
<td>0.501</td>
<td>1.058</td>
<td>0.239</td>
<td>1.17</td>
<td>1.803</td>
</tr>
</tbody>
</table>

- For \(X_q=0.6 Xd\)

Table IV.4: \(IF\) as function of \(P\), \(Q\), \(Ub\) and \(X_p\)

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>Ub</th>
<th>(X_p)</th>
<th>(IF_{final})</th>
<th>(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.013</td>
<td>0.096</td>
<td>0.997</td>
<td>0.207</td>
<td>1.17</td>
<td>1.777</td>
</tr>
<tr>
<td>0.011</td>
<td>0.564</td>
<td>1.054</td>
<td>0.186</td>
<td>1.17</td>
<td>1.761</td>
</tr>
<tr>
<td>0.228</td>
<td>0.083</td>
<td>1.009</td>
<td>0.086</td>
<td>1.16</td>
<td>1.694</td>
</tr>
<tr>
<td>0.228</td>
<td>0.466</td>
<td>1.055</td>
<td>0.189</td>
<td>1.17</td>
<td>1.763</td>
</tr>
<tr>
<td>0.444</td>
<td>0.130</td>
<td>1.009</td>
<td>0.254</td>
<td>1.17</td>
<td>1.816</td>
</tr>
<tr>
<td>0.440</td>
<td>0.461</td>
<td>1.052</td>
<td>0.207</td>
<td>1.17</td>
<td>1.777</td>
</tr>
<tr>
<td>0.846</td>
<td>0.121</td>
<td>1.008</td>
<td>0.334</td>
<td>1.18</td>
<td>1.892</td>
</tr>
<tr>
<td>0.845</td>
<td>0.501</td>
<td>1.058</td>
<td>0.239</td>
<td>1.17</td>
<td>1.803</td>
</tr>
</tbody>
</table>
For $X_q = 0.7 \times X_d$

Table IV.5: $IF$ as function of $P$, $Q$, $Ub$ and $X_p$

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>Ub</th>
<th>$X_p$, $IF_{Final}$</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.013</td>
<td>0.096</td>
<td>0.997</td>
<td>0.207, 1.17, 1.777, 1.192</td>
<td>1.68, 1.288</td>
</tr>
<tr>
<td>0.011</td>
<td>0.564</td>
<td>1.054</td>
<td>0.186, 1.17, 1.761, 1.191</td>
<td>1.667, 1.285</td>
</tr>
<tr>
<td>0.228</td>
<td>0.083</td>
<td>1.009</td>
<td>0.038, 1.16, 1.668, 1.182</td>
<td>1.593, 1.269</td>
</tr>
<tr>
<td>0.228</td>
<td>0.466</td>
<td>1.055</td>
<td>0.187, 1.17, 1.761, 1.191</td>
<td>1.668, 1.285</td>
</tr>
<tr>
<td>0.444</td>
<td>0.130</td>
<td>1.009</td>
<td>0.185, 1.17, 1.761, 1.191</td>
<td>1.667, 1.287</td>
</tr>
<tr>
<td>0.440</td>
<td>0.461</td>
<td>1.052</td>
<td>0.198, 1.17, 1.771, 1.192</td>
<td>1.674, 1.287</td>
</tr>
<tr>
<td>0.846</td>
<td>0.121</td>
<td>1.008</td>
<td>0.23, 1.17, 1.795, 1.194</td>
<td>1.694, 1.291</td>
</tr>
<tr>
<td>0.845</td>
<td>0.501</td>
<td>1.058</td>
<td>0.22, 1.17, 1.787, 1.193</td>
<td>1.688, 1.289</td>
</tr>
</tbody>
</table>
V– Conclusions

From this thesis work, I can conclude the following:

- In a synchronous generator (unsaturated case) the real power ($P$) is higher than that of a cylindrical rotor generator and it occurs at $\delta$ less than 90°. This makes the generator respond quickly to changes in shaft torque.

- If the field excitation ($E_f$) is reduced to zero, the salient pole generator still develop power (torque) because of the saliency of the rotor structure which is not the case in a cylindrical rotor synchronous generator where $P=0$ for $E_f=0$.

- When $E_f=0$ and for $\delta=45^\circ$, the real power ($P$) in synchronous generator reaches its maximum. We can also notice that as the real power ($P$) increases the excitation voltage ($E_f$) increases.

- For $IF$ calculations and in the MATLAB program for synchronous generator (saturated case), $P$, and $Q$, $U_b$ should be kept in a certain range in order to account for saturation. For example, $U_b$ shall always be higher than 0.988 pu.

- For small value of $X_p=0.047$, the relative error is quite large since $X_p$ is proportional to $U_p$ that could not be accurate in the low saturated interval.

- Comparison of the field measurements with the theoretical results shows small differences.

- For $X_q\approx0.5$, 0.6, 0.7$X_d$ the results show a considerable difference with the measurements.

- $X_q\approx0.65X_d$ could be the best approximation.
Appendix A References

Appendix B MATLAB CODE

1) MATLAB code for If calculation (unsaturated case).

%Main Program

%Base values quantities
Un=9.8; %Nominal voltage in kV
Sn=60;  %Nominal complex power in MVA
In=Sn/Un/sqrt(3);  %Nominal current in KA
pf=0.85;  %Power factor
Zb=Un^2/Sn;  %Base Impedance

Xd=0.89;  %d axis synchronous reactance
Xq=0.65*0.89;  %q axis synchronous reactance

%At an operating point

Ub=10.586/Un;
P=0.8247;
Q=0.5778;

Efcal(Xd,Xq,P,Q,Ub)

%Function EfFinal

function EfFinal=Efcal(Xd,Xq,P,Q,Ub)

%Efcal Calculate the field voltage(i.e the field current since it is in pu and on the airgap line)
%and draw the phasor diagram,different power plots,and resultant power for different Ef of a salient
%pole synchronous machine(unsaturated case)
%Efcal(Xd,Xq,P,Q,Ub)
%The first two input arguments are respectively, the d,and q axis synchronous reactances
%The third and the fourth input arguments correspond to the real and reactive power(MW,Mvar)respectively
%The fifth input argument is the terminal voltage(KV)
%All values are expressed in the per unit system to ease the calculations
%A menu will be displayed where you have to choose:Value for Ef ,Phasor Diagram,different power plots,
%resultant power for different Ef

Cosfi=P/sqrt(P^2+Q^2);
phio=acos(Cosfi);
Iao=sqrt(P^2+Q^2)/Ub;
delta=atan(Iao*Xq*Cosfi)/(Ub+Iao*Xq*sin(phio));  % Power angle

a=(Ub/Xd)*sin(delta);
b=((Ub^2+(Xd-Xq))/2*Xd*Xq)*sin(2*delta);
c=(Ub/Xd)*cos(delta);
d=Ub^2*((sin(delta))^2/Xq)+(cos(delta))^2/Xd);
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\[ p = [a^2 + c^2 \ 2a\cdot b - 2c\cdot d \ b^2 + d^2 - P^2 - Q^2]; \]

for \( k = 1:4; \)

\[ k = \text{menu('CHOOSE','Value for If','Phasor Diagram','different power plots','resultant power for different Ef');} \]

if \( k = 1; \)

\[ E_{fn} = \text{roots}(p); \]
\[ E_{fn} = E_{fn}(1) \]

elseif \( k = 2; \)

\[ I_d = \text{abs}(I_{ao})\cdot \sin(\phi_{io} + \delta); \quad \% \text{Stator current in the d axis} \]
\[ I_q = \text{abs}(I_{ao})\cdot \cos(\phi_{io} + \delta); \quad \% \text{Stator current in the q axis} \]
\[ E_f = (U_b\cdot \cos(\delta) - j\cdot \sin(\delta)\cdot U_b) + I_d\cdot X_d + j\cdot I_q\cdot X_q; \]

\[ V_t = (U_b\cdot \cos(\delta) - j\cdot \sin(\delta)\cdot U_b) + I_d\cdot X_d + j\cdot I_q\cdot X_q; \]

\[ x_1 = [0 \ \text{real}(V_t)]; \]
\[ y_1 = [0 \ \text{imag}(V_t)]; \]
\[ \text{plot}(x_1, y_1, 'm-.'); \]
\[ \text{hold on} \]
\[ x_2 = [\text{real}(V_t) \ I_d\cdot X_d + \text{real}(V_t)]; \]
\[ y_2 = [\text{imag}(V_t) \ \text{imag}(V_t)]; \]
\[ \text{plot}(x_2, y_2, 'r--'); \]
\[ \text{hold off} \]
\[ x_3 = [I_d\cdot X_d + \text{real}(V_t) \ I_d\cdot X_d + \text{real}(V_t)]; \]
\[ y_3 = [\text{imag}(V_t) \ 0]; \]
\[ \text{plot}(x_3, y_3, 'b:'); \]
\[ \text{hold off} \]
\[ \text{compass}(E_f, 'g-'); \]
\[ \text{hold off} \]
\[ \text{xlabel('Real axis');} \]
\[ \text{ylabel('Imaginary axis');} \]
\[ \text{title('Phasor diagram for a salient pole synchronous generator');} \]
\[ h = \text{legend('m-.','V_t','r--','IdXd','b:','jIqXq','g-','Ef');} \]

elseif \( k = 3; \)

\[ I_d = \text{abs}(I_{ao})\cdot \sin(\phi_{io} + \delta); \quad \% \text{Stator current in the d axis} \]
\[ I_q = \text{abs}(I_{ao})\cdot \cos(\phi_{io} + \delta); \quad \% \text{Stator current in the q axis} \]
\[ E_f = (U_b\cdot \cos(\delta) - j\cdot \sin(\delta)\cdot U_b) + I_d\cdot X_d + j\cdot I_q\cdot X_q; \]
\[ \text{deltan} = 0:.04:pi; \]
\[ E_{ff} = \text{abs}(E_f); \]
\[ a_n = (U_b\cdot X_d)*\sin(\text{deltan}); \]
\[ b_n = ((U_b^2\cdot (X_d - X_q))/(2\cdot X_d\cdot X_q))\cdot \sin(2\cdot \text{deltan}); \]
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\[
P_f = a_n \cdot E_f; \quad \text{%Power due to field excitation}
P_r = b_n; \quad \text{%Power due to saliency}
P = P_r + P_f; \quad \text{%Resultant power}
\]

\[
\text{plot}(deltan*180/\pi, P_f, 'r', deltan*180/\pi, P_r, 'g', deltan*180/\pi, P, 'b')
\]

grid on;
xlabel('Power angle in degree')
ylabel('Resultant power, Power due to excitation, and Power due to saliency')
title('Different Power curves of a salient pole synchronous generator versus their power angle')
h=legend('r-', 'P_f', 'g-', 'P_r', 'b-', 'P');

else \( k = 4; \)

deltan = 0:0.04:pi; \quad \text{%Power angle}
an = (U_b/X_d)*sin(deltan);
bn = ((U_b^2*(X_d-X_q))/(2*X_d*X_q))*sin(2*deltan);

for \( E_f = 0:0.5:1.5 \)
P_f = (E_f) \cdot a_n; \quad \text{%Power due to field excitation}
P_r = b_n; \quad \text{%Power due to saliency}
P = P_r + P_f; \quad \text{%Resultant power}
\text{plot}(deltan*180/\pi, P, 'r-')
\text{hold off;}
\text{hold on;}
\text{end}

grid on
xlabel('Power angle in degree')
ylabel('Resultant power')
title('Resultant power versus the power angle for different field voltages- \( E_f = 0, 0.5, 1, 1.5 \)pu')
gtext('Ef=0');
gtext('Ef=0.5');
gtext('Ef=1');
gtext('Ef=1.5');
end
end

2) MATLAB code for If calculation (saturated case)

\text{load charbel} \quad \text{% Load data file}
\text{load PQU} \quad \text{% Load values for P, Q and U}
\text{for nr = 102} \quad \text{% 1:length(data)}
\text{clc}
\text{fo = optimset; \quad \text{% Default parameters for numerical passing of TK}}
\text{fo = optimset(fo,'TolX',0.05,'Disp','off'); \quad \text{% Change of defaults}}
\text{TK = data(nr).TK; \quad \text{% TK(:,1) is If in A and TK(:,2) is Ug in kV}}
\text{TK = [TK(:,2) TK(:,1)]/1000; \quad \text{% TK(:,1) is If in A and TK(:,2) is Ug in kV}}
\text{KK = data(nr).KK; \quad \text{% KK(:,1) is If in A and KK(:,2) is Ig in A}}

31
SVK

Methods for determination of the excitation current in synchronous generators

\[ n_1 = TK(:,1); \quad \text{Field current for TK} \]
\[ n_2 = TK(:,2)/\text{data(nr).Un}; \quad \text{TK voltage in pu} \]

% **************************Calculation of TK **************************

if \( n_2(1) > 0.2 \) % Lowerest point under 0.2 pu
\[ n_1 = [0 \ n_1']'; \quad \text{Add a fictive zero with 0.5 \% hysteresis} \]
end

\[ n_3 = \max([\text{find}(n_2 < 0.75)' \ 1]); \quad \text{Find first point below 0.75 pu voltage} \]
\[ \text{if } n_2(n_3) < 0.5 \quad \text{First point under 0.5 pu (maybe too low?)} \]
\[ \quad \text{[n}_4 \ n_3] = \min(\text{abs}(n_2-0.65)); \quad \text{Find nearest point to 0.7 pu} \]
end

% % Percent LK (LuftgapsKurva) is slope of airgapline (A If at 1 pu voltage).
% % LK calculation from Ug/If for points between 0.4 and highest unsaturated point
% % Weighted with Ug, (higher weight at higher voltage). Factor 1.05 är en empirical korrection.

\[ \text{LK} = 1.02*\text{mean}(n_1(n_4+1:n_3))/\text{mean}(n_2(n_4+1:n_3)); \quad \text{Calculate slope with accordance to hysteresis (offset)} \]

else

\[ \text{LK} = 1.02*n_1(n_3)/n_2(n_3); \quad \text{Airgapline is only derived from highest point (bad TK)} \]
end

% % Exclude hysteresis

% IF=(x(1)+ug+max(0,(ug-0.6)).^x(3)*x(4))*x(2)
% X(1)=offset, x(2)=LK, x(3)=exponent x(4)=faktor på exponetiella delen

\[ x = \text{fminsearch('tkcalc',[0 LK 3 4],f0,n2,n1)}; \]
\[ \text{if } x(1)>0.01 \ | x(1)<-0.03 \quad \text{disp('Check hysteresis')} \]
\[ \text{elseif } x(2)/LK>1.1 \ | x(2)/LK<0.9 \quad \text{disp('Check airgap line')} \]
\[ \text{elseif } x(3)>4.6 \ | x(3)<1.7 \ | x(4)>8 \ | x(4)<0.5 \quad \text{disp('Check Curve')} \]
end
\[ x(1)=0; \quad \text{Exclude hysteresis} \]

sat1 = ((x(1)+1+max(0,(1-0.6)).^x(3)*x(4))*x(2)-(x(1)+1)*x(2))/(x(1)+1)*x(2)); % Saturation at 1.0 pu
sat2 = ((x(1)+1.2+max(0,(1.2-0.6)).^x(3)*x(4))*x(2)-(x(1)+1.2)*x(2))/((x(1)+1.2)*x(2)); % Saturation 1.2 pu
pu = (x(1)+1)*x(2); % The pu for field current is:
Methods for determination of the excitation current in synchronous generators

\%disp(\{num2str(n) 9 num2str(LK) 9 num2str(x(1)) 9 num2str(x(2)) 9 num2str(x(3)) 9 num2str(x(4))\})

n5=0:0.01:ceil(max(1.35,max(n2))*100)*0.01;

\% Curve at TK with passing function and air gap line
plot(((x(1)+n5+max(0,(n5-0.6)).^x(3)*x(4))/x(2))/pu,n5,'r',n1/pu,n2,'or',[x(1)*x(2) (x(1)+1.35)*x(2)]/pu,[0 1.35],'g')
set(gcf,'PaperOrient','landscape','PaperPos',\{0.634517 0.634517 28.4084 19.715\});
hold on

\%Base values quantities
Un=data(nr).Un;
Sn=data(nr).Sn;
In=Sn/Un/sqrt(3);
Zb=Un^2/Sn;

\% ********************************* Calculation of KK *********************************

n1=KK(:,1);
n2=KK(:,2)./(Sn/Un/sqrt(3))/1000;
if n2(1)>0.2
    n1=[0 n1']';
    n2=[0.005 n2']';
end
n3=polyfit(n1,n2,1);
if n4>1
    disp('Check KK:s linearity.')
end

\% ******************* Engineering units is 1 in pu-base
clear n1 n2 n3 n4 n5 ans fo
clear TK KK Sn Un In LK Zb
Ubp=data(nr).Ubp/data(nr).Un;
Ibp=data(nr).Ibp/data(nr).In;
Cosfibp=data(nr).Cosfibp;
Hfpb=1000*data(nr).Ibp/pu;
Cosfin=data(nr).Cosfin;
Methods for determination of the excitation current in synchronous generators

disp(['Nr ' num2str(nr) ' ' data(nr).Name ']
if pu=' sprintf('%0.6g',(x(1)+1)*x(2))];

% Available data for calculation, x is for airgap & saturation function
%Cosfin,Cosfibp,Ubp,Ifbp,Xd,Xds,Kc,sat1,sat2,pu,x

disp(['Cosfin = ' num2str(Cosfin)])
disp(['Cosfibp = ' num2str(Cosfibp)])
disp(['Ubp = ' num2str(Ubp)])
disp(['Ifbp = ' num2str(Ifbp)])
disp(['Xd = ' num2str(Xd)])
disp(['Xds = ' num2str(Xds)])
h=legend('r-','OCC','g-','airgapline','b-','SCC');

% ************************************************** Start of exjob, all data is in pu **************************************************

phin=acos(Cosfin); % Power factor angle
Xq=0.6*Xd; % Q axis synchronous reactance

% At an operating point

phipb=acos(Cosfibp); % Power factor angle
Iao=Ibp*(Cosfibp-j*sin(phipb)); % Change stator current to complex value
P=Ubp*Ibp*cosfibp; % Real output power (pu)
Q=Ubp*Ibp*sin(phipb); % Reactive output power (pu)
delta=atan((Ibp*Xq*Cosfibp)/(Ubp+Ibp*Xq*sin(phipb))); % Power angle
Id=Ibp*sin(phipb+delta); % Stator current in the d axis
Iq=Ibp*cos(phipb+delta); % Stator current in the q axis

% abs below is only to guaranty real value due to numeric noise
Efus=abs(Ubp*(cos(delta)-j*sin(delta)) + Id*Xq+j*Iq*Xq); % Calculate unsaturated field current at operating point

% Example how to calculate If, where x is parameters from earlier passing:

IFS=Ifbp-Efus; % Saturation part of If (test with different Ef)
Up=0.6+exp((log(IFS)-log(x(4)))/x(3)); % Voltage behind Potier reactance
mag=sqrt(Up^2-(Ubp*Cosfibp)^2)-Ubp*sin(phipb); % Magnitude value for Up-Ubp
Xpp=mag/Ibp % Potier reactance

for n=1:8

Pn=F(n,1);
Qn=F(n,2);
Ub=G(n,1);
end
Ifcall(Xd,Xq,Xpp,x,Pn,Qn,Ub)
end
%Function IFFinal

function IFFinal=Ifcall(Xd,Xq,Xpp,x,Pn,Qn,Ub)
load PQU

%Ifcall Calculate the field current and draw the phasor diagram of a salient pole synchronous machine
%(Saturated case)
%Ifcall (Xd, Xq,Xp,x,Pn,Qn,Ub)
%The first three input arguments are respectively, the d,q axis synchronous reactances and the Potier reactance
%The fourth input argument correspond to x(3)=exponent x(4)=factor
%The fifth and the sixth input arguments correspond to the real and reactive power (MW, Mvar) respectively
%The seventh input argument is the terminal voltage (KV)
%A menu will be displayed where you have to choose: Value for If or Phasor Diagram

for n=1:8

    Pn=F(n,1);
    Qn=F(n,2);
    Ub=G(n,1);

    Cosfi=Pn/sqrt(Pn^2+Qn^2);
    phi=acos(Cosfi);
    Ib=sqrt(Pn^2+Qn^2)/Ub;
    delta=atan((Ib*Xq*Cosfi)/(Ub+Ib*Xq*sin(phi))); % Power angle

    Iao=Ib*(Cosfi-j*sin(phi));
    Id=Ib*sin(phi+delta); % Stator current in the d axis
    Iq=Ib*cos(phi+delta); % Stator current in the q axis

    Up=abs(Ub+j*Iao*Xpp); %Voltage behind Potier reactance

    a=(Ub/Xd)*sin(delta);
    b=(((Ub^2.*(Xd-Xq))/(2.*Xd*Xq))*sin(2.*delta));
    c=(Ub/Xd)*cos(delta);
    d=Ub^2*((sin(delta))^2/Xq)+(cos(delta))^2/Xd;

    p=[a^2+c^2 2*a*b-2*c*d b^2+d^2-Pn^2-Qn^2];
    Efn=roots(p);
    Ef_final(n)=Efn(1);
    IFFinal(n)=Ef_final(n)+max(0,(Up-0.6)).^x(3)*x(4);
end

for k=1:2;

    k = menu('CHOOSE','Value for If','Phasor Diagram');
Methods for determination of the excitation current in synchronous generators

```matlab
if k==1;
    IFFinal(n)=Ef_final(n)+max(0,(Up-0.6)).^x(3)*x(4)
elseif k==2;
    %Phasor Diagram
    Ef=(Ub*cos(delta)-j*sin(delta)*Ub)+Id*(Xd)+j*Iq*(Xq);
    Ea=(Ub*cos(delta)-j*sin(delta)*Ub);
    x1=[0 real(Ea)];
    y1=[0 imag(Ea)];
    plot(x1,y1,'m-.');
    hold on;
    x2=[real(Ea) Id*(Xd)+real(Ea)];
    y2=[imag(Ea) imag(Ea)];
    plot(x2,y2,'r--');
    hold off;
    hold on;
    x3=[Id*(Xd)+real(Ea) Id*(Xd)+real(Ea)];
    y3=[imag(Ea) 0];
    plot(x3,y3,'b:');
    hold off;
    hold on;
    compass(Ef,'g-');
    hold off;
    xlabel('Real axis');
    ylabel('Imaginary axis');
    title('Phasor diagram for a salient pole synchronous generator')
    h=legend('m-.','Ea','r--','jIdXd','b:','jIqXq','g-','Ef');
end
end
```
## Appendix C List of symbols

### Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MATLAB</td>
<td>Matrix Laboratory</td>
</tr>
<tr>
<td>Pu</td>
<td>Per Unit</td>
</tr>
<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronics Engineering</td>
</tr>
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</table>

### Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>Magnetic flux density (Weber/m², Tesla)</td>
</tr>
<tr>
<td>d-q-0</td>
<td>direct, quadrature and zero axis</td>
</tr>
<tr>
<td>Ef</td>
<td>Field voltage (Volts)</td>
</tr>
<tr>
<td>Efn</td>
<td>Unsaturated field voltage (Volts)</td>
</tr>
<tr>
<td>f</td>
<td>electrical frequency (Hertz-Hz)</td>
</tr>
<tr>
<td>F</td>
<td>Magneto Motive Force (Ampere-A)</td>
</tr>
<tr>
<td>Fa</td>
<td>Armature MMF (Ampere-A)</td>
</tr>
<tr>
<td>Fd</td>
<td>Magneto Motive Force in the d-direction (Ampere-A)</td>
</tr>
<tr>
<td>Ff</td>
<td>Field MMF (Ampere-A)</td>
</tr>
<tr>
<td>Fq</td>
<td>Magneto Motive Force in the q-direction (Ampere-A)</td>
</tr>
<tr>
<td>H</td>
<td>Magnetic field intensity (Ampere/meter-A/m)</td>
</tr>
<tr>
<td>I</td>
<td>Current (Ampere-A)</td>
</tr>
<tr>
<td>Ia</td>
<td>Armature current (Ampere-A)</td>
</tr>
<tr>
<td>Ib</td>
<td>Armature current at operating point (Ampere-A)</td>
</tr>
<tr>
<td>Id</td>
<td>Armature current in the d-direction (Ampere-A)</td>
</tr>
<tr>
<td>IF</td>
<td>Field current (Ampere-A)</td>
</tr>
<tr>
<td>IFS</td>
<td>Saturation increment current (Ampere-A)</td>
</tr>
<tr>
<td>IFU</td>
<td>Field current at an operating point (Ampere-A)</td>
</tr>
<tr>
<td>Iq</td>
<td>Armature current in the q-direction (Ampere-A)</td>
</tr>
<tr>
<td>In</td>
<td>Nominal current (Ampere-A)</td>
</tr>
<tr>
<td>K</td>
<td>Winding factor</td>
</tr>
<tr>
<td>n</td>
<td>Rotor speed (Round per minute-rpm)</td>
</tr>
<tr>
<td>N</td>
<td>Number of turns in each phase</td>
</tr>
<tr>
<td>p</td>
<td>number of poles</td>
</tr>
<tr>
<td>P</td>
<td>Real power (Watts-W)</td>
</tr>
<tr>
<td>Pf</td>
<td>Real power due to field excitation (Watts-W)</td>
</tr>
<tr>
<td>Pr</td>
<td>Real power due to saliency (Watts-W)</td>
</tr>
<tr>
<td>pf</td>
<td>Power factor (degree)</td>
</tr>
<tr>
<td>pfo</td>
<td>Power factor at operating point (degree)</td>
</tr>
<tr>
<td>phio</td>
<td>Power factor angle at operating point (degree)</td>
</tr>
<tr>
<td>Q</td>
<td>Reactive power (VAR)</td>
</tr>
<tr>
<td>Sn</td>
<td>Nominal complex power (VA)</td>
</tr>
<tr>
<td>Ub</td>
<td>Terminal stator voltage (Volts-V)</td>
</tr>
<tr>
<td>Ubp</td>
<td>Terminal stator voltage (operating point) (Volts-V)</td>
</tr>
<tr>
<td>Un</td>
<td>Nominal voltage (Volts-V)</td>
</tr>
<tr>
<td>Uc</td>
<td>Voltage back to Potier reactance (Volts-V)</td>
</tr>
</tbody>
</table>
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vt</td>
<td>Terminal voltage</td>
<td>(Volts-V)</td>
</tr>
<tr>
<td>X</td>
<td>Reactance</td>
<td>(Henry-H)</td>
</tr>
<tr>
<td>Xad</td>
<td>d-axis armature reactance</td>
<td>(Henry-H)</td>
</tr>
<tr>
<td>Xal</td>
<td>Leakage reactance</td>
<td>(Henry-H)</td>
</tr>
<tr>
<td>Xaq</td>
<td>q-axis armature reactance</td>
<td>(Henry-H)</td>
</tr>
<tr>
<td>Xar</td>
<td>Armature reactance</td>
<td>(Henry-H)</td>
</tr>
<tr>
<td>Xd</td>
<td>d-axis synchronous reactance</td>
<td>(Henry-H)</td>
</tr>
<tr>
<td>Xq</td>
<td>q-axis synchronous reactance</td>
<td>(Henry-H)</td>
</tr>
<tr>
<td>Xp</td>
<td>Potier reactance</td>
<td>(Henry-H)</td>
</tr>
<tr>
<td>Xs</td>
<td>Synchronous reactance</td>
<td>(Henry-H)</td>
</tr>
<tr>
<td>Zb</td>
<td>Base impedance</td>
<td>(Ohm-Ω)</td>
</tr>
<tr>
<td>δ</td>
<td>Power or load angle</td>
<td>(degree)</td>
</tr>
<tr>
<td>φ</td>
<td>Power factor angle</td>
<td>(degree)</td>
</tr>
<tr>
<td>φad</td>
<td>Armature flux in the d-direction</td>
<td>(Weber)</td>
</tr>
<tr>
<td>φaq</td>
<td>Armature flux in the q-direction</td>
<td>(Weber)</td>
</tr>
<tr>
<td>φar</td>
<td>Armature flux</td>
<td>(Weber)</td>
</tr>
<tr>
<td>φf</td>
<td>Field flux</td>
<td>(Weber)</td>
</tr>
<tr>
<td>μ</td>
<td>Permeability of the medium</td>
<td>(Henry/meter)</td>
</tr>
<tr>
<td>ℜ</td>
<td>Reluctance</td>
<td>(Ampere/Weber)</td>
</tr>
</tbody>
</table>